HW 1: Algebra/Intro Closes Tonight!

HW 2: 2.2 Separable eqns, slope fields

2.2: Separable Differential Equations

Entry Task: *(Motivation)*

Implicitly differentiate $x^2 + y^3 = 8$ and solve for $\frac{dy}{dx}$.

Idea: Separate… integrate both sides.

Entry Task continued:

Find the *explicit* solution for

$$\frac{dy}{dx} = \frac{-2x}{3y^2}$$

with $y(0) = 2$.

Separable Differential Equations

A **separable** differential equation can be written as:

$$\frac{dy}{dx} = f(x)g(y).$$

(or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ or $\frac{dy}{dx} = \frac{g(y)}{f(x)}$.)

Example: Find the explicit solution to

$$\frac{dy}{dx} = -3xy$$

with $y(0) = 4$.

You do: Find the explicit solution to

$$\frac{dy}{dx} = 2xy^2$$

with $y(2) = \frac{1}{5}$.

What if the initial condition was $y(2) = 0$?

Observations:

A 1st order differential equation can have:

1. No Solution

2. Infinitely many solutions (one "parameter" or "free constant", initial conditions not given)

3. A unique solutions (initial conditions given)

In a class on the theory of differential equations you would talk about how this is more detail (conditions on the differential equations in order for a solution to exist and be unique). Read 2.4 and ask me questions if you are interested in learning more.

Example: Find an *implicit* solution to

$$\frac{dy}{dx} = \frac{3x + 1}{5y^4 - y}$$

with $y(2) = 1$.

Example: Find the general *explicit* solution to

$$2\frac{dy}{dx} = 3x^2(y^2 - 1)$$

Example:
A town currently has 2100 people
- The birth/death rate is proportional to the population size with a relative growth rate of $k = 0.03$.
- In addition, 100 people/year are immigrating into the city from elsewhere.
Let $P(t)$ be the number of people in the city in t years from now.
Find $P(t)$.

Example:

Consider

$$\frac{dy}{dx} = 3x - y$$

This is NOT separable. It is "linear" and we will discuss a method on Wednesday for this type.

But if you leave this course, you may encounter a method called "change of variable" to "fix" a problem like this. Let's try one.

Assume I tell you to let $v = 3x - y$

Find

$\frac{dv}{dx} =$

This new equation is separable!! Solve it, then rewrite your final answer in terms of y and x.